CHAPTER 2

PHYSICAL OPTICS– DIFFRACTION

INTRODUCTION

The wave nature of light is further confirmed by the optical phenomenon of diffraction. The word 'diffraction' is derived from the Latin word *diffractus* which means to break to pieces. It is common experience that waves bend around obstacles placed in their path. When light waves encounter an obstacle, they bend round the edges of the obstacle. This bending is predominant when the size of the obstacle is comparable to the wavelength of light. The bending of light waves around the edge of an obstacle is diffraction. It was first observed by Gremaldy.

2.1 DIFFRACTION

As shown in Fig. 2.1, when light falls on an obstacle then the corresponding geometrical shadow on the screen should be completely dark. In practice, the geometrical shadow consists of bright and dark fringes. These fringes are due to the superimposition of bended light waves around the corners of an obstacle. The amount of bending depends upon the size of an obstacle and wavelength of light.

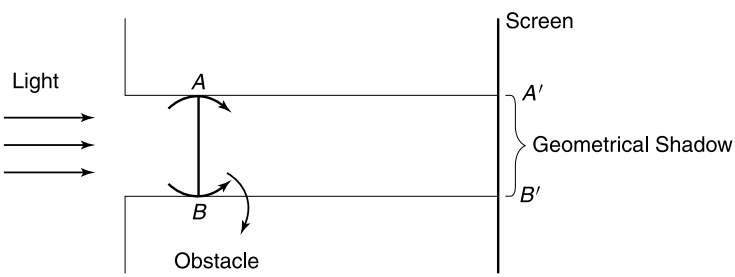


Figure 2.1 Diffraction

When light falls on an obstacle whose size is comparable with the wavelength of light then light bends around the edges or corners of an obstacle and enters

into the geometrical shadow. This bending of light is known as *diffraction*. The bright and dark fringes in the shadow forms a diffraction pattern.

The diffraction phenomena are broadly classified into two types:

Fresnel diffraction

To study diffraction, there should be a light source, obstacle and screen. In this class of diffraction, the source and screen are placed at finite distances from the obstacle. To study this diffraction, lenses are not necessary as the source and screen are at a finite distance. This diffraction can be studied in the direction of propagation of light. The incident wave fronts are either spherical or cylindrical.

Fraunhofer diffraction

In this class of diffraction, the source and screen are placed at infinite distances from the obstacle. Due to the above fact, we need lenses to study the diffraction. This diffraction can be studied in any direction. In this case, the incident wave front is plane.

2.1.1 Comparison between Fresnel's and Fraunhofer's Diffraction

Fraunhofer diffraction	*Fresnel's diffraction*
1. For diffraction to occur, the light source and screen are at infinite distance from the obstacle.	1. For diffraction to occur, the light source and screen are at finite distance from the obstacle.
2. To study diffraction, lenses are necessary.	2. No lenses are necessary to study the diffraction.
3. Study of the diffraction is easy.	3. Study of the diffraction is complicated.
4. Diffraction can be studied in any direction of propagation of light.	4. Diffraction can be studied only in the direction of propagation of light.
5. In this case, the incident wavefronts are plane.	5. In this case, the incident wavefronts are either spherical or cylindrical.

2.1.2 Difference between Interference and Diffraction

When two or more light waves superimpose then it causes intensity modifications in the resultant light wave, which causes the interference phenomenon. In diffraction, the bended light waves superimpose and cause intensity variations in the form of a diffraction pattern. The diffraction phenomenon also involves interference effect. Let us study the differences between interference and diffraction phenomena.

Interference	*Diffraction*
1. It is due to superposition of two source different wavefronts originating from two coherent sources.	1. It is due to superposition of secondary wavelets originating from the different parts of the same wavefront.

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2. Interference bands are of equal width.	2. Diffraction bands decrease in their widths as the order increases.
3. All the bright fringes are of the same intensity.	3. The bright fringes are of varying intensity.
4. All the dark fringes have zero intensity.	4. The intensity of dark fringes is not zero.

2.2 FRAUNHOFER DIFFRACTION AT SINGLE SLIT

Consider a slit *AB* of width '*e*'. Let a plane wave front *WW′* of monochromatic light of wavelength λ propagating normally towards the slit is incident on it. The diffracted light through the slit is focussed by means of a convex lens on a screen placed in the focal plane of the lens. According to Huygen–Fresnel, every point on the wave front in the plane of the slit is a source of secondary wavelets, which spread out to the right in all directions. These wavelets travelling normal to the slit, i.e., along the direction OP_0 are brought to focus at P_0 by the lens. Thus, P_0 is a bright central image. The secondary wavelets travelling at an angle θ with the normal are focussed at a point P_1 on the screen. Depending on path difference, the point P_1 may have maximum or minimum intensities. To find intensity at P_1, let us draw the normal *AC* from *A* to the light ray at *B*.

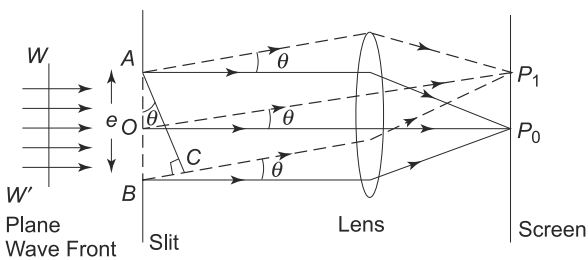


Figure 2.2 Fraunhofer diffraction—single slit

The path difference between the wavelets from *A* and *B* in the direction θ is given by

$$\text{Path difference} = BC = AB \sin\theta = e \sin\theta$$

$$\text{Corresponding phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} e \sin\theta$$

Let the width of the slit be divided into *n* equal parts and the amplitude of the wave from each part is '*a*'. The phase difference between any two successive waves from these parts would be

$$\frac{1}{n}[\text{Total phase}] = \frac{1}{n}\left[\frac{2\pi}{\lambda} e \sin\theta\right] = d \text{ (say)}$$

Using the method of vector addition of amplitudes, the resultant amplitude R is given by

$$R = \frac{a \sin nd/2}{\sin d/2}$$

$$= \frac{a \sin (\pi e \sin \theta/\lambda)}{\sin (\pi e \sin \theta/n\lambda)}$$

$$= a \frac{\sin \alpha}{\sin \alpha/n} \quad \text{where } \alpha = \pi e \sin \theta/\lambda$$

$$= a \frac{\sin \alpha}{\alpha/n} \quad (\because \alpha/n \text{ is very small})$$

$$= n \frac{a \sin \alpha}{\alpha} \quad (\because na = A)$$

$$= A \frac{\sin \alpha}{\alpha}$$

Intensity
$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \tag{1}$$

2.2.1 Principal Maximum

The resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

I will be maximum, when the value of R is maximum. For maximum value of R, the negative terms must vanish, i.e., $\alpha = 0$

$$\frac{\pi e \sin \theta}{\lambda} = 0$$

$$\sin \theta = 0$$

$$\theta = 0 \tag{2}$$

Then
$$R = A$$

$$I_{max} = R^2 = A^2 \tag{3}$$

The condition $\theta = 0$ means that the maximum intensity is formed at P_0 and is known as *principal maximum*.

2.2.2 Minimum Intensity Positions

I will be minimum, when $\sin \alpha = 0$

$$\alpha = \pm \pi, \pm 2\pi, \pm 3\pi$$

$$\alpha = \pm m\pi$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

$$e \sin \theta = \pm m\lambda \tag{4}$$

where $m = 1, 2, 3.....$

Thus, we obtain the points of minimum intensity on either side of the principal maximum. For $m = 0$, $\sin \theta = 0$, which corresponds to principal maximum.

2.2.3 Secondary Maxima

In between these minima, we get secondary maxima. The positions can be obtained by differentiating the expression of I w.r.t. a and equating to zero. We get

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin \alpha}{\alpha}\right)^2\right] = 0$$

$$A^2 \cdot \frac{2 \sin \alpha}{\alpha} \cdot \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

$$\text{either } \sin \alpha = 0 \quad \text{or} \quad \alpha \cos \alpha - \sin \alpha = 0$$

$$\sin \alpha = 0 \quad \text{gives positions of minima.}$$

Hence the positions of secondary maxima are given by

$$\alpha \cos \alpha - \sin \alpha = 0$$

$$\alpha = \tan \alpha \tag{5}$$

The values of α satisfying the above equation are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan \alpha$ on the same graph. The points of intersection of the two curves gives the values of α which satisfy the above equation. The plots of $y = \alpha$ and $y = \tan \alpha$ are shown in Fig. 2.3.

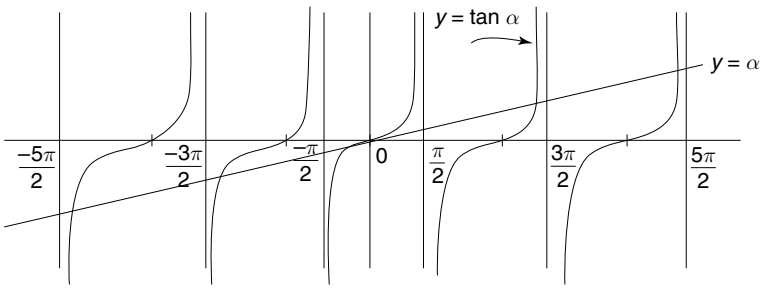


Figure 2.3 Plots of $y = \alpha$ and $y = \tan \alpha$

The points of intersections are

$$\alpha = 0, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, ...$$

Substituting the above values in Eq. (1), we get the intensities in various maxima.

$$\alpha = 0, I_0 = A^2 \quad \text{(principal maximum)}$$

$$\alpha = \frac{3\pi}{2}, \quad I_1 = A^2 \left[\frac{\sin(3\pi/2)}{3\pi/2}\right]^2 \approx \frac{A^2}{22} \quad (1^{st} \text{ secondary maximum})$$

$$\alpha = \frac{5\pi}{2}, \quad I_2 = A^2 \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 \approx \frac{A^2}{62} \quad (2^{nd} \text{ secondary maximum})$$

and so on.

From the above expressions, it is evident that most of the incident light is concentrated in the principal maximum and the rest is redistributed into the secondary maxima.

2.2.4 Intensity Distribution

The variation of I w.r.t. α is shown in Fig. 2.4. The diffraction pattern consists of a central principal maximum for $\alpha = 0$. There are secondary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$. Between secondary maxima, there are minima at positions $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

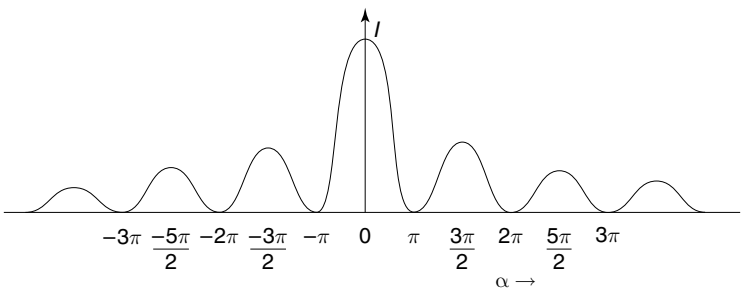


Figure 2.4 Intensity distribution

2.3 FRAUNHOFER DIFFRACTION AT DOUBLE SLIT

Let S_1 and S_2 be double slits of equal widths e and separated by a distance d. The distance between the middle points of the two slits is $(e + d)$. A monochromatic light of wavelength λ is incident normally on the two slits. The light diffracted from these slits is focussed by a lens on the screen placed in the focal plane of the lens. The diffraction at two slits is the combination of diffraction as well as interference, i.e., the pattern on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superimposed. When a plane wavefront is incident normally on two slits, the secondary wavelets from the slits travel uniformly in all directions. The wavelets travelling in the direction of incident light come to a focus at P_0 while the wavelets travelling in a direction making an angle θ, come to focus at P_1.

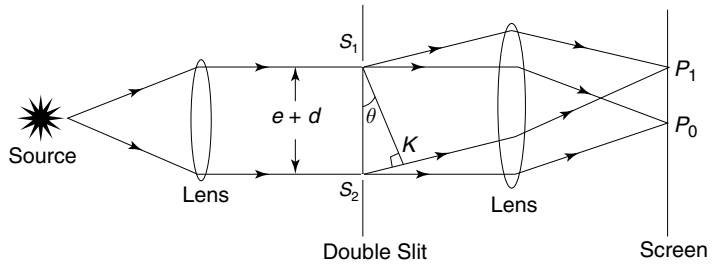


Figure 2.5 Fraunhofer diffraction—double slit

From the study of diffraction due to single slit, the resultant amplitude

$$= \frac{A \sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi e \sin \theta}{\lambda}.$$

Since we use double slit, from each slit we get a wavelet of amplitude $\frac{A \sin \alpha}{\alpha}$ in a direction θ. These two wavelets interfere and meet at a point P_1 on the screen. To calculate the path difference between the wavelets, let us draw a normal S_1K to the wavelet through S_2.

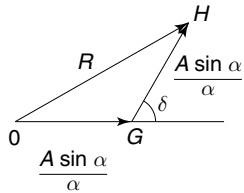


Figure 2.6 Vector addition method

$$\text{Path difference} = S_2K$$
$$= (e + d) \sin \theta$$

$$\text{Phase difference} \quad \delta = \frac{2\pi}{\lambda} (e + d) \sin \theta \tag{6}$$

To find the resultant amplitude at P_1 we use vector addition method (Fig. 2.6) in which the two sides of a triangle are represented by the amplitudes through S_1 and S_2. The third side gives the resultant amplitude.

From the figure,

$$(OH)^2 = (OG)^2 + (GH)^2 + 2\,(OG)(GH) \cos \delta$$

$$R^2 = \left(\frac{A \sin \alpha}{\alpha}\right)^2 + \left(\frac{A \sin \alpha}{\alpha}\right)^2 + 2 \left(\frac{A \sin \alpha}{\alpha}\right)\left(\frac{A \sin \alpha}{\alpha}\right) \cos \delta$$

$$= A^2 \frac{\sin^2 \alpha}{\alpha^2} [2 + 2 \cos \delta]$$

$$= 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 (1 + \cos \delta)$$

$$= 2 \left(\frac{A \sin \alpha}{\alpha}\right)^2 (1 + 2 \cos^2 \delta/2 - 1)$$

$$R^2 = 4A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \cos^2 \left[\frac{\pi (e + d) \sin \theta}{\lambda}\right]$$

Let $$\beta = \frac{\pi (e + d) \sin \theta}{\lambda}$$

$$R^2 = 4A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \cos^2\beta$$

The resultant intensity $I = R^2 = 4A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \cos^2\beta$ (7)

From the above expression, it is clear that the resultant intensity is the product of two factors, i.e.,

1. $A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2$ which represents the diffraction pattern due to a single slit
2. $\cos^2\beta$ which gives the interference pattern due to wavelets from double slits

The resultant intensity is due to both Diffraction and Interference effects.

2.3.1 Diffraction Effect

The diffraction term $A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2$ gives the principal maximum at the centre of the screen with alternate minima and secondary maxima of decreasing intensity. We get principal maximum for $\theta = 0$. We get minima for $\sin \alpha = 0$

$$\alpha = \pm m\pi, \quad \text{where } m = 1, 2, 3, ...$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

$$e \sin \theta = \pm m\lambda$$

The positions of secondary maxima occurs for

$$\alpha = \frac{\pm 3\pi}{2}, \frac{\pm 5\pi}{2}, \frac{\pm 7\pi}{2}, ...$$

2.3.2 Interference Effect

The interference term $\cos^2 \beta$ gives the equidistant bright and dark fringes.

The maxima will occur for $\cos^2 \beta = 1$

$$\beta = \pm n\pi, \quad \text{where } n = 0, 1, 2, 3, ...$$

$$\beta = 0, \pm \pi, \pm 2\pi, \pm 3\pi ...$$

$$\frac{\pi (e + d) \sin \theta}{\lambda} = \pm n\pi$$

$$(e + d) \sin \theta = \pm n\lambda$$

The minima will occur for $\cos^2 \beta = 0$

$$\beta = \pm (2n + 1)\pi/2 \quad \text{where } n = 0, 1, 2, 3, ...$$

$$(e + d) \sin \theta = \pm (2n + 1)\, \lambda/2$$

2.3.3 Intensity Distribution

Figure 2.7 (a) and (b) represents the intensity variations due to diffractions as well as interference effect individually. When both effects are combined then we get the resultant intensity variation (Fig. 2.7 (c)). From the figure, it is clear that the resultant minima are not equal to zero, still they have some minimum intensity due to interference effect.

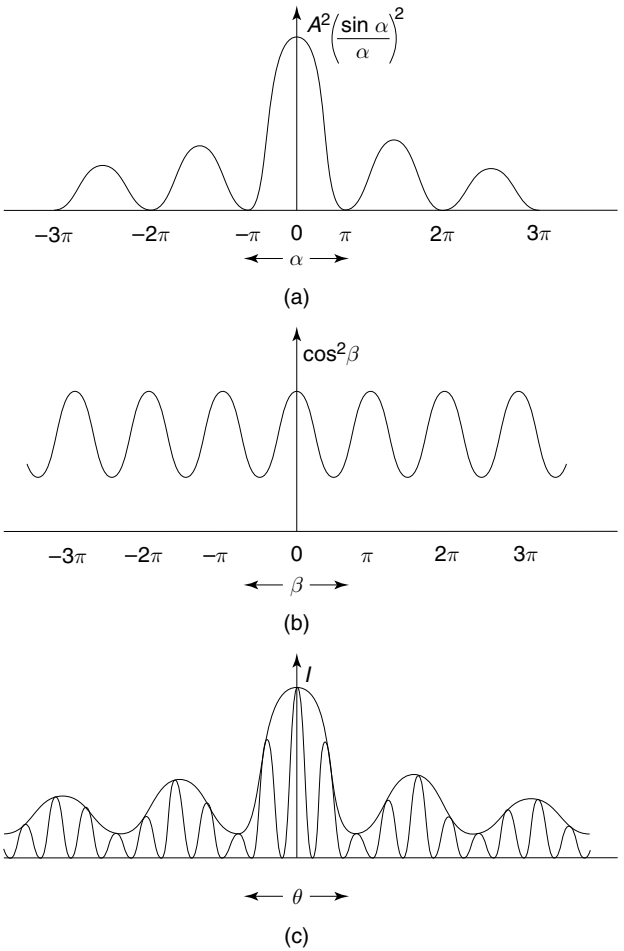


Figure 2.7 (a) Diffraction effect, (b) Interference effect, (c) Resultant intensity

2.4 DIFFRACTION GRATING

An arrangement which consists of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular intervals. Now gratings are constructed

by ruling equidistant parallel lines on a transparent material such as glass with a fine diamond point. The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit (Fig. 2.8).

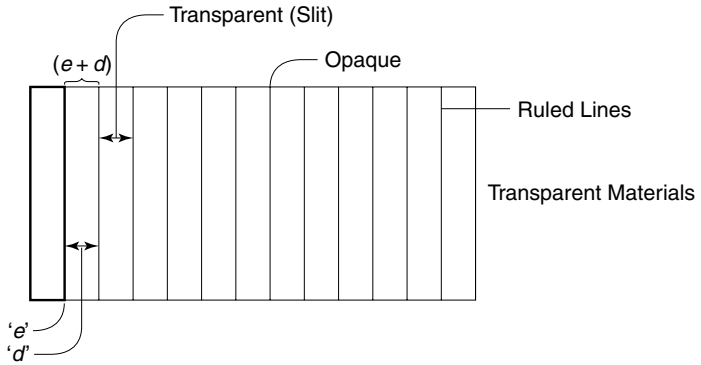


Figure 2.8 Diffraction grating

Commercial gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. Solution of cellulose acetate is poured on the ruled surface and allowed to dry to form a thin film, detachable from the surface. These impressions of a grating are preserved by mounting the film between two glass sheets.

Let '*e*' be the width of the line and '*d*' be the width of the slit. Then $(e + d)$ is known as *grating element*. If '*N*' is the number of lines per inch on the grating then

$$N\,(e + d) = 1'' = 2.54 \text{ cm}$$

$$e + d = \frac{2.54}{N} \text{ cm}$$

There will be nearly 30,000 lines per inch of a grating. Due to the above fact, the width of the slit is very narrow and is comparable to wavelengths of light. When light falls on the grating, the light gets diffracted through each slit. As a result, both diffraction and interference of diffracted light gets enhanced and forms a diffraction pattern. This pattern is known as *diffraction spectrum*.

2.5 FRAUNHOFER DIFFRACTION DUE TO GRATING (N PARALLEL SLITS)

Consider a grating *AB* consisting of *N* parallel slits with grating element $(e + d)$. Let a plane wavefront $\omega\omega'$ of monochromatic light of wavelength λ be incident normally on the grating. The diffracted light through the *N* parallel slits is focussed by means of a convex lens on a screen placed in the focal plane of the lens. The secondary wavelets travelling normal to the slit or along the direction OP_0 are brought to focus at P_0 by the lens. The point P_0 will be the central

maximum. The secondary wavelets travelling at an angle θ with the direction of the incident light are focussed at a point P_1 on the screen.

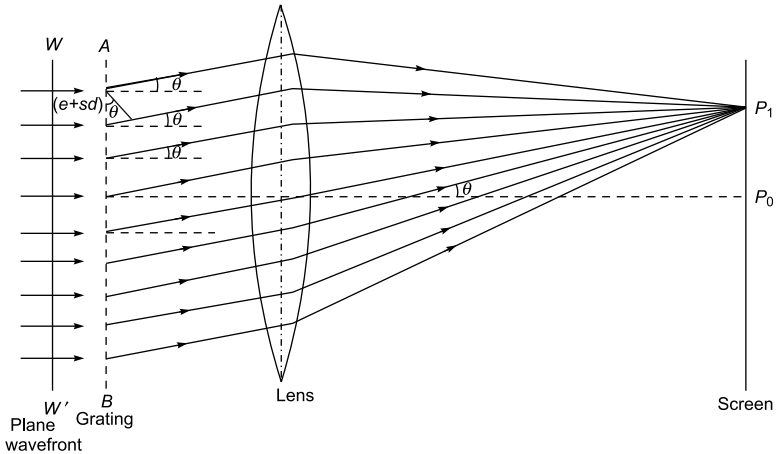


Figure 2.9 Fraunhofer diffraction-grating

Since the grating consists of N parallel slits from each slit are light wave emerges and we have N waves through the slits. From the concepts of single slit, we get N waves of amplitude $\frac{A \sin \alpha}{\alpha}$ where $\alpha = \frac{\pi e \sin \theta}{N}$ emerges in the direction of θ.

The path difference between the waves emerging through two successive slits

$$= (e + d) \sin \theta$$

The corresponding phase difference

$$= \frac{2\pi}{\lambda} (e + d) \sin \theta = 2 \beta$$

From the method of vector additions of amplitudes, the resultant amplitude is given by

$$R = \frac{a \sin nd/2}{\sin d/2}$$

For the grating $a = \frac{A \sin \alpha}{\alpha}$, $n = N$ and $d = 2 \beta$

$$R = \frac{A \sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$$

and

$$I = R^2 = \left(\frac{A \sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The factor $\left(\frac{A \sin \alpha}{\alpha} \right)^2$ gives the diffraction effect due to single slit and $\left(\frac{\sin N\beta}{\sin \beta} \right)^2$ represents the combined effects of interference and diffraction due to all the slits of grating.

2.5.1 Principal Maxima

I will be maximum when $\sin \beta = 0$

$$\beta = \pm n\pi \, (n = 0, 1, 2, 3\ldots.)$$

At the same time $\sin N\beta = 0$, then $\dfrac{\sin N\beta}{\sin \beta}$ becomes indeterminate.

By using L Hospital's rule, we set

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to \pm n\pi} \frac{\dfrac{d}{d\beta}(\sin N\beta)}{\dfrac{d}{d\beta}(\sin \beta)}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N \cos N\beta}{\cos \beta} = \pm N$$

$$\lim_{\beta \to \pm n\pi} \left(\frac{\sin N\beta}{\sin \beta} \right)^2 = N^2$$

$$I = \left(\frac{A \sin \alpha}{\sin \alpha} \right)^2 N^2$$

The above intensity represents principal maximum and are obtained for $\beta = \pm n\pi$

$$\frac{\pi}{\lambda}(e + d) \sin \theta = \pm n\pi$$

$$(e + d) \sin \theta = \pm n\lambda$$

where $n = 0, 1, 2, 3$ ….

$n = 0$ corresponds to zero order principal maximum. For $n = 1, 2, 3,$ …. we get first, second, third …. order principal maxima. The ± sign shows that there are two principal maxima of the same order lying on either side of zero order maximum.

Minima A series of minima occur, when

$$\sin N\beta = 0 \text{ but } \sin \beta \neq 0$$

For minima $\sin N\beta = 0$ or $N\beta = \pm m\pi$

$$N \frac{\pi}{\lambda}(e + d) \sin \theta = \pm m\pi.$$

$$N(e + d) \sin \theta = \pm m\lambda,$$

where m has all integral values except 0, N, $2N$, … nN, because for these values $\sin \beta$ becomes zero and we get principal maxima. Thus, $m = 1, 2, 3 \ldots (N - 1)$. Hence, there are adjacent principal maxima.

2.5.2 Secondary maxima

As there are $(N-1)$ minima between two adjacent principal maxima, there must be $(N-2)$ other maxima between two principal maxima. These are known as secondary maxima. To find out the position of these secondary maxima. Let us differentiate the intensity equation with respect to β and then equate it to zero. Thus

$$\frac{dI}{d\beta} = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \cdot 2\left(\frac{\sin N\beta}{\sin\beta}\right) \times \left[\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

or
$$N\cos N\beta\sin\beta - \sin N\beta\cos\beta = 0$$
$$N\tan\beta = \tan N\beta$$

The roots of this equation other than those for which $\beta = \pm n\pi$ (principal maxima) give the positions of secondary maxima. To find out the value of $(\sin^2 N\beta/\sin^2\beta)$ from equation $N\tan\beta = \tan N\beta$, we make use of the triangle shown in Fig. 2.10 It gives

$$\sin N\beta = \frac{N}{\sqrt{N^2 + \cot^2\beta}}$$

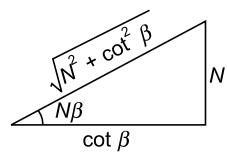


Figure 2.10

$$\therefore \quad \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2 + \cot^2\beta) \times \sin^2\beta}$$

$$= \frac{N^2}{N^2 + \sin^2\beta + \cos^2\beta}$$

$$= \frac{N^2}{1 + (N^2 - 1)\sin^2\beta}$$

$$\therefore \quad \frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}}$$

$$= \frac{1}{1 + (N^2 - 1)\sin^2\beta}$$

As N increases, the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.

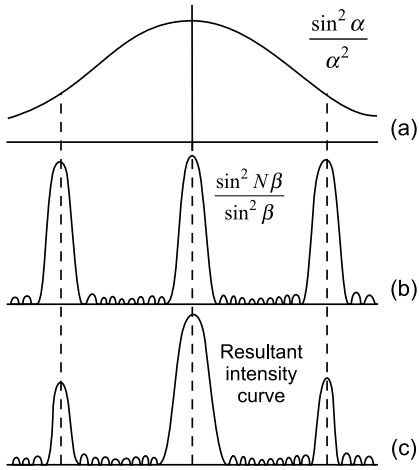


Figure 2.11

Figure 2.11 (a) and (b) show the variation of intensity due to the factors $\sin^2\alpha/\alpha^2$ and $\sin^2 N\beta/\sin^2\beta$ respectively. The resultant intensity is shown in Fig. 2.3(c).

SOLVED PROBLEMS

1. A plane transmission grating having 4250 lines per cm is illuminated with a sodium light normally. In the second-order spectrum, the spectral lines are deviated by 30°. What is the wavelength of the spectral line?

Given data Number of lines in the grating $N = 4250$ lines/cm

Angle of diffraction $\theta = 30°$

Order of spectrum $n = 2$

Solution $(e + d) \sin\theta = n\lambda$ (i)

$$\text{Each slit width } (e + d) = \frac{1}{N} = \frac{1}{4250} = 2.353 \times 10^{-4} \text{ cm}$$

Then substituting the value of e in Eq. (i)

$$2.353 \times 10^{-4} \times \sin 30° = 2\lambda$$
$$2.353 \times 10^{-4} \times 0.5 = 2\lambda$$
$$1.1765 \times 10^{-4} = 2\lambda$$
$$\Rightarrow \quad \lambda = 0.5882 \times 10^{-4} \text{ cm}$$
$$= 5.882 \times 10^{-5} \text{ cm}$$
$$\lambda = 5882 \text{ Å}$$

2. A source of light having a wavelength of 600 nm is incident on a slit with a width of 1 μm. Find the angular separation between the first-order minima and central maxima of either side.

Given data Wavelength of the light, $\lambda = 600\,\text{nm} = 600 \times 10^{-9}$ m

Width of the slit, $e + d = 1 \times 10^{-6}$ m

Order $n = 1$

Solution $(e + d) \sin\theta = n\lambda \Rightarrow \sin\theta = \dfrac{n\lambda}{(e+d)} \Rightarrow \theta = \sin^{-1}\left(\dfrac{n\lambda}{e+d}\right)$

$$\theta = \sin^{-1}\left(\frac{1 \times 600 \times 10^{-9}}{1 \times 10^{-6}}\right) = \sin^{-1}(0.6) = 36°52'$$

Angular separation is 36°52′.

3. A plane grating having 10520 lines per cm is illuminated with light having a wavelength of 5×10^{-5} cm at normal incidence. How many orders are visible in the grating spectra?

Given data Number of lines in the grating $N = 10520$ lines/cm

Width of each slit, $(e + d) = \dfrac{1}{N}$

Wavelength of the light, $\lambda = 5 \times 10^{-5}$ cm

Angle $\theta = 90°$

Solution $(e + d) \sin\theta = n\lambda$

where $(e + d) = \dfrac{1}{N} = \dfrac{1}{10520} = 9.506 \times 10^{-5}$

$$n = \frac{e \sin\theta}{\lambda}$$

$$n = \frac{9.506 \times 10^{-5} \times \sin 90°}{5 \times 10^{-5}} = 1.901$$

$$n = 2$$

In this grating, two orders can be seen.

4. A grating has 6000 lines/cm. Find the angular separation between two wavelengths of 500 nm and 510 nm in the 3rd order.

Given data Number of lines in the grating,

$$N = \frac{1}{e+d}\ 6000 \text{ lines/cm}$$

$$= 6000 \times 10^2 \text{ lines/m}$$

Order $n = 3$

Solution We know that $(e + d) \sin \theta = n\lambda$

$$\sin \theta = \frac{n\lambda}{e + d} = nN\lambda$$

For first wavelength, $\lambda_1 = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$

$$\sin \theta_1 = 3 \times 6000 \times 10^2 \times 500 \times 10^{-9}$$

$$\sin \theta_1 = 0.9$$

$$\theta_1 = \sin^{-1}(0.9) = 64°9'$$

Similarly, $\lambda_2 = 510 \text{ nm} = 510 \times 10^{-9} \text{ m}$

$$\sin \theta_2 = 3 \times 6000 \times 10^2 \times 510 \times 10^{-9}$$

$$= 0.918$$

$$\theta_2 = \sin^{-1}(0.918)$$

$$\theta_2 = 66°38'$$

Angular separation between two wavelengths in third order

$$= \theta_2 - \theta_1$$

$$= 66°38' - 64°9'$$

$$= 2°29'$$

5. Find the highest order that can be seen with a grating having 15000 lines/inches. The wavelength of the light used is 600 nm.

Given data Wavelength of the given light $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$

Number of lines in the grating $N = \frac{1}{e + d} = 15000$ lines/inch

$$= 15000/2.54 \text{ lines/cm}$$

$$= 5906 \text{ lines/cm}$$

$$= 5906 \times 10^2 \text{ lines/m}$$

Solution We know that

$$n_{max} = \frac{e + d}{\lambda} = \frac{1}{N\lambda}$$

$$n_{max} = \frac{1}{5906 \times 10^2 \times 600 \times 10^{-9}}$$

$$n_{max} = 2.822$$

$\therefore$ the highest order that can be seen is 2.

6. For a grating, the angle of diffraction for the second order principal maximum for the wavelength 5×10^{-5} cm is 30°. Find the number of lines per cm of the grating.

Given data Wavelength of light $\lambda = 5 \times 10^{-5}$ cm

Diffraction angle $\theta = 30°$

Order $n = 2$

Solution $(e + d) \sin \theta = n\lambda$

$$e + d = \frac{n\lambda}{\sin \theta} = \frac{2 \times 5 \times 10^{-5}}{\sin 30^\circ} = \frac{2 \times 5 \times 10^{-5}}{0.5}$$

$$= 10^{-3} \text{ cm}$$

$\therefore$ Number of lines per cm, $N = \dfrac{1}{e + d} = 10^3$.

7. The first diffraction minima due to a single slit diffraction is at $\theta = 30^\circ$ for a light of wavelength 5000 Å. Find the width of the slit.

Given data Diffraction angle $\theta = 30^\circ$

Wavelength of light $\lambda = 5000$ Å $= 5000 \times 10^{-8}$ cm

Order $m = 1$

Solution Condition for minima due to single slit is

$$e \sin \theta = m\lambda$$

$$e = \frac{m\lambda}{\sin \theta} = \frac{1 \times 5000 \times 10^{-8}}{\sin 30^\circ} = \frac{1 \times 5000 \times 10^{-8}}{0.5}$$

$$= 10 \times 10^{-5} \text{ cm}$$

$\therefore$ width of the slit $= 10 \times 10^{-5}$ cm

8. A monochromatic light of wavelength 6.56×10^{-7} m incidents normally on a grating of 2 cms wide. The first odder spectrum is produced at an angle of 18°14′ from the normal. Calculate the total number of lines in the grating.

Solution $\sin \theta = Nn\lambda$

$$N = \frac{\sin \theta}{n\lambda}$$

but $$N = \frac{\text{total no. of lines}}{\text{width}} = \frac{M}{0.02}$$

Hence, $$M = \frac{0.02 \sin \theta}{n\lambda} = \frac{0.02 \times \sin 18^\circ 14'}{1 \times 6.56 \times 10^{-7}} = 9540$$

9. A parallel beam of sodium light of wavelength 5890 Å is incident on a thin glass plate of refractive index 1.5, such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection?

Given data Wavelength of light $\lambda = 5890$ A°

Refractive index of glass plate $\mu = 1.5$

Angle of refraction $r = 60^\circ$

Solution The plate will appear dark in the reflected system if

$$2\mu t \cos r = n\lambda$$

for the smallest thickness $n = 1$

$$\text{Hence } t = \frac{\lambda}{2\mu \cos r} = \frac{5890 \times 10^{-10}}{2 \times 1.5 \times \cos 60} = 3927 \times 10^{-10}$$
$$= 3927 \text{ Å}.$$

10. A single slit is illuminated by white light. The first minimum for red light of wavelength $\lambda = 6500$ Å falls at an angle $\theta = 30^\circ$. What is the value of slit width?

Given data Wavelength of red colour $\lambda = 6500 \text{ Å} = 6500 \times 10^{-10}$ m.
angle $\theta = 30^\circ$, slit width $2d = ?$

Solution $2d \sin \theta = \lambda$

$$2d = \frac{\lambda}{\sin \theta} = \frac{6500 \times 10^{-10}}{1/2} = 13000 \times 10^{-10} = 13000 \text{ Å}$$

11. Light from sodium vapour lamp is normally falls on a grating of 2 cm having 10000 lines. Find the angular separation of the two lines of sodium of wavelengths 5890 and 5896 Å in the first order spectrum?

Given data Wavelengths of sodium lamps $\lambda_1 = 5890$ Å
$\lambda_2 = 5896$ Å
Width of the Grating = 2 cm, order $n = 1$

Solution $N = \frac{10000}{2} = 5000$ lines/cm $= 5000 \times 10^2$ lines/m.

We know that $\sin \theta = Nn\lambda$

For $\lambda_1 \rightarrow \sin \theta_1 = 1 \times 5 \times 10^5 \times 5890 \times 10^{-10} = 0.29450$

$$\theta_1 = 17.1275^\circ$$

for λ_2 $\sin \theta_2 = nN\lambda_2$
$= 1 \times 5 \times 10^5 \times 5896 \times 10^{-10}$ m
$= 0.29480$
$\theta_2 = 17.1458^\circ$
$\theta_2 - \theta_1 = 0.018^\circ$

MULTIPLE CHOICE QUESTIONS

1. Diffraction phenomena are usually divided into _____ classes.
   (a) one (b) two
   (c) three (d) four

2. In which experiment the screen and sources are at finite distance?
   (a) Fraunhofer diffraction (b) Fresnel's diffraction
   (c) Both (a) and (b) (d) None of these
3. In Fraunhofer's diffraction, incident light waves have _____ type of wave front.
   (a) circular (b) spherical
   (c) cylindrical (d) plane
4. In which diffraction are lenses required?
   (a) Fraunhofer diffraction (b) Fresnel's diffraction
   (c) Young's diffraction (d) Newton's diffraction
5. In single-slit experiment, if the red colour is replaced by blue then _____
   (a) the diffraction pattern becomes narrower and crowded together
   (b) the diffraction bands become wider
   (c) the diffraction pattern does not change
   (d) the diffraction pattern disappears
6. Diffraction grating has a
   (a) large number of equidistant slits
   (b) only one slit
   (c) large number of random distant slits
   (d) circular slit
7. In a diffraction grating, the condition for principal maxima is
   (a) $e \sin \theta = n\lambda$ (b) $(e + d) \sin \theta = n\lambda$
   (c) $d \sin \theta = n\lambda$ (d) $\sin \theta = n\lambda$
8. What are the characteristics of grating spectra?
   (a) Spectral lines are almost straight and outer sharp.
   (b) The spectral lines are more and more dispersed as order increases.
   (c) It is situated symmetrically on both sides of zero order image.
   (d) All of the above
9. In Fraunhofer diffraction the wavefront undergoing diffraction has to be
   (a) cylindrical (b) spherical
   (c) elliptical (d) plane
10. The penetration of waves into the regions of the geometrical shadow is
    (a) dispersion (b) diffraction
    (c) interference (d) polarization
11. In single-slit diffraction, the first diffraction minima is observed at an angle of 30°, when a light having a wavelength of 500 nm is used. The width of the slit is
    (a) 1.25×10^{-5} cm (b) 10×10^{-5} cm
    (c) 5×10^{-5} cm (d) 2.5×10^{-15} cm
12. In a single-slit experiment if the slit width is reduced,
    (a) the fringes become narrower
    (b) the fringes becomes brighter

(c) the fringes become wider

(d) the colour of the fringes change

13. The diffraction pattern of a single slit consists of

(a) narrow bright band at the centre with alternate dark and bright bands of equal intensity on either side

(b) wider bright band at the centre with alternate dark and bright bands of equal intensity on either side

(c) wider dark band at the centre with alternate bright and dark bands on either side

(d) wider and brighter band at the centre with alternate dark and bright bands of decreasing intensity on either side

14. In diffraction due to double slit, we observe

(a) wider diffraction bands and within that narrower interference fringes

(b) wider interference fringes and narrower diffraction bands

(c) interference and diffraction fringes of equal width

(d) diffraction pattern due to both the slits independently

15. A parallel beam of monochromatic light falls normally on a plane diffraction grating having 5000 lines/cm. A second-order spectral line is diffracted through an angle of 30°. The wavelength of light is

(a) 5×10^{-6} cm (b) 5×10^{-7} cm

(c) 5×10^{-5} cm (d) 5×10^{-4} cm

16. When white light is incident on a diffraction grating, the light diffracted more will be

(a) blue (b) yellow

(c) violet (d) red

17. Maximum number of orders possible with a grating is

(a) independent of grating element

(b) inversely proportional to grating element

(c) directly proportional to grating element

(d) directly proportional to wavelength

18. When white light is incident on a grating, the light diffracted less will be

(a) blue (b) yellow

(c) violet (d) red

19. For a grating, if the width of the grating element is less than twice the wavelength of light, then the possible number of diffraction orders are

(a) one (b) two

(c) three (d) four

20. Diffraction effect is predominant when

(a) size of the obstacle is less than the wavelength of light

(b) size of the obstacle is nearly equal to the wavelength of light

(c) size of the obstacle is greater than the wavelength of light

(d) None of the above

Answers

1. (b)	2. (b)	3. (d)	4. (a)	5. (a)	6. (a)
7. (b)	8. (d)	9. (d)	10. (b)	11. (b)	12. (c)
13. (d)	14. (a)	15. (c)	16. (d)	17. (c)	18. (c)
19. (a)	20. (b)				

Fill in the Blanks

1. The bending of light around the corners of obstacles is called _____.
2. In Frensel's diffraction _____ source is used.
3. In diffraction pattern fringes have _____ width.
4. With decreasing of slit width in single slit, the width of the fringes _____.
5. In a grating, the combined width of a ruling and a slit is called _____.
6. Points on successive slits Separated by a distance equal to the grating element is called _____.
7. In Fraunhofer diffraction at a plane transmission grating, when white light source is used, the angle of diffraction for violet is _____ than that of red.

Answers to Fill in the blanks

1. diffraction	2. point	3. unequal
4. increases	5. Grating element	6. Corresponding points
7. lesser		

True or False

1. For diffraction to occur, the size of the obstacle must be comparable with wavelength. (T/F)
2. Converging lense is used to focus parallel rays in Fresnel diffraction. (T/F)
3. The source and Screen are at infinite distance from the obstacle producing Fraunhofer diffraction. (T/F)
4. Diffraction is due to superposition of secondary waves originating from different parts of the same wave front. (T/F)
5. In diffraction all bright fringes have same intensity. (T/F)
6. In diffraction intensity of dark fringes is not zero. (T/F)
7. In Fraunhofer diffraction at a double slit, the angular separation between two consecutive maxima decreases if the slits are narrower and closer. (T/F)

Answers to True or False

1. T	2. F	3. T	4. T	5. F	6. T
7. T					

REVIEW QUESTIONS

Short Answer Questions

1. What is diffraction?
2. Why lenses are necessary for the study of Fraunhofer diffraction?
3. Mention any two differences between interference and diffraction.
4. Derive the relationship between the phase difference and path difference of two light waves.
5. Mention the conditions to get principal maximum and minimum intensity positions due to single slit by Fraunhofer diffraction.
6. Explain why interference effects are significant along with the diffraction effect for double slits Fraunhofer diffraction?
7. What is diffraction grating?
8. Explain why interference effects are more significant along with the diffraction effect in grating.

Essay Type Questions

1. Distinguish between Fraunhofer and Fresnel's diffraction.
2. Distinguish between interference and diffraction.
3. Describe Fraunhofer diffraction due to single slit.
4. Describe Fraunhofer diffraction due to double slits.
5. Describe Fraunhofer diffraction due to grating.